Power Electronics

Exercise: Power Dissipation and Cooling of Power Electronic Devices

2012
1 Theory

1.1 Losses

The power losses occurring in power electronic devices can be classified as following:

1.1.1 Conduction losses

A voltage drop occurs across each conducting device. Considering diodes and thyristors a voltage-current characteristic as in fig. 1.1 can be assumed. In order to simplify calculations the second part of the conduction characteristic is approximated with a straight line. This line intersects with the voltage axis in the point \( v = V_S \). \( V_S \) is called threshold voltage. The differential resistance

\[
r_D = \frac{\Delta v}{\Delta i}
\]

(1.1)
can be calculated from the line’s slope. Hence the approximated straight line can be described with the following equation:

\[
v = V_S + r_D i
\]

(1.2)

For the conduction losses follows (time dependent):

\[
p_D = vi = V_S i + r_D i^2
\]

(1.3)

If \( v \) and \( i \) are periodic, the heat output which has to be dissipated is

\[
P_D = \frac{1}{T} \int_0^T p_D dt = V_S \cdot \frac{1}{T} \int_0^T i dt + r_D \cdot \frac{1}{T} \int_0^T i^2 dt
\]

(1.4)

If the temporal average value

\[
I_{AV} = \frac{1}{T} \int_0^T i dt
\]

(1.5)

and the RMS value

\[
I_{RMS} = \sqrt{\frac{1}{T} \int_0^T i^2 dt}
\]

(1.6)
\[ P_D = V_s I_{AV} + r_D I_{RMS}^2 \]  

results. This equation can also be used for the calculation of the conduction losses of a bipolar transistor or an IGBT. The bulk resistance \( R_{on} \) between drain and source of a field effect transistor is approximately constant. Hence, the conduction losses for these devices are

\[ P_D = I_{RMS}^2 R_{on} \]

### 1.1.2 Blocking losses

If the semiconductors are in blocking mode, a quite small blocking current is flowing, while a high voltage is between the devices’ power terminals. In order to estimate the blocking losses, the trend of the blocking voltage \( v_R(t) \) has to be known. The trend of the blocking current \( i_R(t) \) can be estimated with the help of an approximation line. Quite often, however, it is enough to assume a constant value \( I_R \) for the blocking current, as it can be seen in fig. 1.2.

![Fig. 1.2: Blocking characteristic of a diode or a thyristor](image)

For the case of a sinusoidal blocking voltage \( v_R(t) = \dot{v}_R \cdot \sin(\omega t) \) the following can be obtained:

\[ P_R = \frac{1}{T} \int_0^T p_R(t) \, dt = \frac{1}{T} I_R \int_0^T v_R(t) \, dt = \frac{1}{\pi} \dot{v}_R I_R \]  

Blocking losses, however, are mostly negligible.

### 1.1.3 Control losses

A control current is necessary to turn a device on or off. The losses which are created by these currents are also mostly negligible.

### 1.1.4 Switching losses

During the transient from blocked to conducting state and vice versa high values of the current and the voltage appear simultaneously for a short time. For this time the losses are very high. Switching losses are proportional to the switching frequency. The turn-on losses can be calculated with the following equation:

\[ W_{on} = \frac{1}{t_0 + t_{on}} \int_{t_0}^{t_0 + t_{on}} p \, dt \]
Turn-off losses can be calculated with

\[ W_{off} = \int_{t_0}^{t_0+t_{off}} p \, dt \]  

(1.11)

If the semiconductor switch is turned on and off with the frequency \( f \), the whole switching losses are given by

\[ P_S = f (W_{on} + W_{off}) \]  

(1.12)

Fig. 1.3: Switching losses at turn-on and turn-off

1.2 Thermal equivalent circuit

1.2.1 Heat conduction

The heat conduction in a compound in which a heat current is led from a part 1 to a part 2 can be described as following:

\[ R_{th} = \frac{\vartheta_1 - \vartheta_2}{P} \]  

(1.13)

\[ R_{th} = \frac{d}{\lambda A} \]  

(1.14)

with

- \( R_{th} \): Thermal resistance \( \left[ \frac{K}{W} \right] \)
- \( \lambda \): Thermal conductivity \( \left[ \frac{W}{Km} \right] \)
- \( A \): Sectional area of the compound perpendicular to the heat current
- \( d \): Thickness of the compound in direction of the heat current

1.2.2 Thermal storage

The energy \( P \, dt \) which is put into the compound will be completely converted to a temperature change \( d\vartheta \):

\[ P = C_{th} \frac{d\vartheta}{dt} = C_{th} \dot{\vartheta} \]  

(1.15)

\[ C_{th} = V \gamma c \]  

(1.16)
with

\[ C_{th} : \text{Heat capacity} \left[ \frac{Ws}{K} \right] \]
\[ V : \text{Volume} \]
\[ \gamma : \text{Specific mass} \]
\[ c : \text{Specific heat capacity} \]

### 1.2.3 Analogy to electrical circuits

Temperature trajectories can be calculated analogously to voltage trajectories in electrical circuits:

- The power loss \( P \) which has to be led away from the compound corresponds to the electric current (heat current).
- The temperatures correspond to the electric potentials or voltages.

### 1.2.4 General thermal equivalent circuit

A general thermal equivalent circuit diagram for the heat transfer and thermal storage with \( n \) compounds is given in fig. 1.4.

![Thermal equivalent circuit diagram](image)

Fig. 1.4: Thermal equivalent circuit diagram

### 1.2.5 Equivalent thermal circuit diagram with partial fractions

For the calculation of the warming of devices the equivalent thermal circuit diagram given in fig. 1.4 is suitable only to a limited extent. For this reason an equivalent series circuit diagram is used which leads to partial fractions. In fig. 1.5 an example is shown: it represents the same circuit as in fig. 1.4 if the values \( R_{th1}, R_{th2}, \ldots R_{thn} \) and \( C_{th1}, C_{th2}, \ldots C_{thn} \) are selected correspondingly.

If a time-varying power is put into point \( A \), the following differential equation can be obtained:

\[ p(t) = \frac{\vartheta_1}{R_{th1}} + C_{th1} \frac{d\vartheta_1}{dt} = \frac{\vartheta_2}{R_{th2}} + C_{th2} \frac{d\vartheta_2}{dt} = \ldots \]  

(1.17)

For the case that a constant power is put into an element which is in thermal equilibrium, i. e. \( p(t) = 0 \ \forall \ t < 0, p(t) = P = \text{const.} \ \forall \ t \geq 0 \), the following temperature profiles can be obtained for the compounds:

\[ \vartheta_i = PR_{thi} \left( 1 - e^{-\frac{t}{\tau_{thi}}} \right), \ i = 1 \ldots n \]  

(1.18)
with

\[ \tau_{thi} = R_{thi} \cdot C_{thi} \]  \hspace{1cm} (1.19)

Hence, the temperature in point \( A \) can be calculated to

\[ \vartheta_A = \sum_{i=1}^{n} \vartheta_i + \vartheta_B \]  \hspace{1cm} (1.20)

For stationary operation with \( p = \text{const.} \), i.e. \( t \gg 4r_{\text{max}} \) the thermal capacitances are negligible.

### 1.2.6 Transient thermal resistances

The thermal equivalent circuit diagram shown in fig. 1.5 can be further simplified by introducing so-called transient thermal resistances. These are defined as follows:

\[ Z_{thi}(t) = R_{thi} \left( 1 - e^{-\frac{t}{\tau_{thi}}} \right) \]  \hspace{1cm} (1.21)

The resulting equivalent circuit diagram with transient thermal resistances can be seen in fig. 1.6.

If equation 1.21 is replaced in 1.18 and the resulting equation again in 1.20, the following coherence can be obtained for the temperature in point \( A \) if transient thermal resistances are used:

\[ \vartheta_A(t) = P \cdot \sum_{i=1}^{n} Z_{thi}(t) + \vartheta_B \]  \hspace{1cm} (1.22)
1.2.7 Operation with time-varying losses

If losses are time-varying, the superposition principle can be applied, i. e. the solutions of the equations 1.17 and 1.20 for the single periods can be added. This principle is shown schematically in fig. 1.7.

Fig. 1.7: Application of the superposition principle for the calculation of time-varying temperature profiles
2 Exercises

2.1 Exercise 1

Half waves of a sinusoidal current are flowing through a diode. The impulses have a duration of \( t_i = 100 \mu s \), the peak amplitude of the impulses is \( i_D = 500 \text{ A} \). The maximum allowed loss power is 150 W. The diode characteristics are: \( U_S = 1.4 \text{ V} \), \( r_D = 0.9 \text{ m}\Omega \).

What is the maximum frequency \( f_p \) of the impulses, considering that the switching loss energy

a) is negligible,

b) is 0.2 Ws for one switching event?

Note: \( \int \sin^2 (ax) \, dx = \frac{x}{2} - \frac{\sin (2ax)}{4a} \)
2.2 Exercise 2

A periodically time-dependent current (period $T$) is flowing through a diode whose characteristics are shown in fig. 2.1:

\[
\begin{align*}
    i &= I_M = 50 \text{ A for } 0 \leq t < \frac{T}{2} \\
    i &= 0 \text{ A for } \frac{T}{2} \leq t < T
\end{align*}
\]

What is the value of the conduction loss power (active power) $P_D$ which is converted into heat within the diode?

![Characteristics of the diode](image)

Fig. 2.1: Characteristics of the diode
2.3 Exercise 3

For a thyristor with heatsink the characteristics shown in fig. 2.2 are effective at the frequency $f = 50 \text{ Hz}$. $Z_{thJC}$ is the inner and $Z_{thCA}$ is the outer transient heat resistance, $\delta$ is the firing angle of the thyristor ($0^\circ \leq \delta \leq 180^\circ$). The maximum allowed temperature in the semiconductor is $\vartheta_J = 115^\circ \text{C}$, the ambient temperature can be assumed to be $\vartheta_A = 45^\circ \text{C}$.

Draw the thermal equivalent circuit.

What is the value of the loss power $P$ of the thyristor, considering that

a) direct current is flowing for $t = 10 \text{ s}$,

b) direct current is flowing in continuous operation,

c) current impulses with $f = 50 \text{ Hz}$ ($\delta = 30^\circ$) are flowing in continuous operation?

![Graphs showing $Z_{thJC}$ and $Z_{thCA}$](image)

**Fig. 2.2:** Inner (left) and outer (right) transient heat resistance of the thyristor at $f = 50 \text{ Hz}$
2.4 Exercise 4

A diode with $V_S = 1.05\,\text{V}$ and a differential resistance $r_D = 0.9\,\text{m}\Omega$ is given. It is flown by a sinusoidal current (only positive half waves) with the peak amplitude $\hat{i} = 300\,\text{A}$.

a) Which value do the conduction losses $P$ within the diode have?

b) Draw the thermal equivalent circuit.

c) For the given continuous operation the diode has the transient thermal resistances $Z_{thJC} = 0.143\,\text{K/W}$ and $Z_{thCA} = 0.6\,\text{K/W}$. The maximum allowed junction temperature is $\vartheta_{Jmax} = 115\,^\circ\text{C}$.

Calculate the maximum allowed coolant temperature $\vartheta_{Amax}$!

d) Which limit $\vartheta_{Cmax}$ has to be set to a temperature monitor that observes the case temperature via a sensor? $\vartheta_{Jmax} = 115\,^\circ\text{C}$ has to be guaranteed.

e) What is the maximum allowed loss power $P'$ if a maximum coolant temperature of $\vartheta_{Amax} = 35\,^\circ\text{C}$ is allowed?

Note: $\int \sin^2(ax)\,dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$
2.5 Exercise 5

The heat transfer between the junction of a diode and its body is given by four thermal $RC$ terms ($R_{thi} \parallel C_{thi}$) which are connected in series. The heat resistances and heat capacitances have the following values:

\[
R_{th1} = 0.019 \, \frac{K}{W}, \quad R_{th2} = 0.033 \, \frac{K}{W}, \quad R_{th3} = 0.222 \, \frac{K}{W}, \quad R_{th4} = 0.068 \, \frac{K}{W} \\
C_{th1} = 0.158 \, \frac{W_s}{K}, \quad C_{th2} = 0.758 \, \frac{W_s}{K}, \quad C_{th3} = 0.468 \, \frac{W_s}{K}, \quad C_{th4} = 14.68 \, \frac{W_s}{K}
\]

The diode is supplied with the power loss impulse which is shown in fig. 2.3.

![Power loss impulse](image)

Fig. 2.3: Power loss impulse

a) Draw the thermal equivalent circuit diagram.

b) Calculate the value of the time constant $\tau_{thi}$ of the transient heat resistances.

c) Calculate the heating of the junction compared to the heating of the body at the end of the impulse.

d) Draw the temperature profile of the junction during the impulse.

e) The diode is mounted on a heatsink for enhanced air cooling. In fig. 2.4 the transient thermal resistance $Z_{thCA}$ of the heatsink including the heat transfer for enhanced air cooling is shown. After which time is, assuming a loss power of 85 W, the junction temperature equal to 80°C if the ambient temperature is 35°C?

f) How big is the allowed loss power $P$ (assumption: $P = \text{const.}$) considering an ambient temperature of 35°C and an allowed junction temperature of 125°C?
Fig. 2.4: Transient thermal resistance of the heatsink for enhanced air cooling
3 Solutions

3.1 Exercise 1

Fig. 3.1 shows the sinusoidal half waves of the current.

![Graph of a sinusoidal current with half-wave peaks at t_i and T_p](image)

Fig. 3.1: Half waves of the sinusoidal current flowing through the diode

The equation for the current $i_D(t)$ can be formulated as follows:

$$i_D(t) = \hat{i}_D \sin \left( \frac{\pi}{t_i} t \right) \text{ for } 0 \leq t \leq t_i$$
$$i_D(t) = 0 \quad \text{for } t_i \leq t \leq T_p$$

The voltage drop across the diode can be calculated to:

$$v_D(t) = V_S + i_D(t) \cdot r_D$$

During a current impulse the energy

$$W_D = \int_{0}^{t_i} v_D(t) \cdot i_D(t) \, dt =$$

$$= \int_{0}^{t_i} V_S \cdot i_D(t) \, dt + \int_{0}^{t_i} r_D \cdot i_D^2(t) \, dt =$$

$$= \int_{0}^{t_i} V_S \cdot \hat{i}_D \sin \left( \frac{\pi}{t_i} t \right) \, dt + \int_{0}^{t_i} r_D \cdot \hat{i}_D^2 \sin^2 \left( \frac{\pi}{t_i} t \right) \, dt =$$

$$= V_S \cdot \hat{i}_D \cdot \left[ \frac{t_i}{\pi} \cos \left( \frac{\pi}{t_i} t \right) \right]_0^{t_i} + r_D \cdot \hat{i}_D^2 \cdot \left[ \frac{t}{2} - \frac{\sin \left( \frac{2\pi}{t_i} t \right)}{\frac{2\pi}{t_i}} \right]_0^{t_i} =$$

$$= V_S \cdot \hat{i}_D \cdot \left[ \frac{t_i}{\pi} + \frac{t_i}{\pi} \right] + r_D \cdot \hat{i}_D^2 \cdot \left[ \frac{t_i}{2} - 0 \right] =$$

$$= V_S \cdot \hat{i}_D \cdot 2 \frac{t_i}{\pi} + r_D \cdot \hat{i}_D^2 \cdot \frac{1}{2} t_i =$$

$$= 55.81 \text{ mWs}$$

is dissipated in the diode. From the maximum allowed loss power the impulse frequencies can be calculated:
a) \( f_p = \frac{P_{\text{max}}}{W_D} = 2.69 \text{ kHz} \)

b) \( f_p = \frac{P_{\text{max}}}{W_D + W_S} = 0.59 \text{ kHz} \)
3.2 Exercise 2

The temporal average value of the current is

\[ I_{AV} = \frac{1}{T} \int_{0}^{T} i \, dt = \frac{1}{T} I_M \frac{T}{2} = \frac{I_M}{2} = \frac{50 \, A}{2} = 25 \, A \]

The RMS value can be calculated to

\[ I_{RMS} = \sqrt{\frac{1}{T} \int_{0}^{T} i^2 \, dt} = \sqrt{\frac{1}{T} I_M^2 \frac{T}{2}} = \frac{I_M}{\sqrt{2}} = \frac{50 \, A}{\sqrt{2}} = 35.4 \, A \]

From fig. 2.1 the threshold voltage can be figured out:

\[ V_S = 0.8 \, V \]

The differential resistance can also be calculated with fig. 2.1:

\[ r_D = \frac{\Delta u}{\Delta i} = \frac{0.2 \, V}{50 \, A} = 4.0 \, m\Omega \]

Hence, the loss power can be calculated:

\[ P = V_S \cdot I_{AV} + r_D \cdot I_{RMS}^2 = 25 \, W \]
3.3 Exercise 3

In fig. 3.2 the thermal equivalent circuit diagram is shown.

![Thermal equivalent circuit diagram of the thyristor with heatsink]

Fig. 3.2: Thermal equivalent circuit diagram of the thyristor with heatsink

a) The transient thermal resistances $Z_{thJC} = 0.6 \frac{K}{W}$ and $Z_{thCA} = 0.4 \frac{K}{W}$ can be taken from fig. 2.2. Hence, the allowed loss power can be calculated to:

$$P = \frac{\theta_J - \theta_A}{Z_{thJC} + Z_{thCA}} = 70 \text{ W}$$

b) In this case the transient thermal resistances are $Z_{thJC} = 0.6 \frac{K}{W}$ and $Z_{thCA} = 1.2 \frac{K}{W}$. The resulting loss power is now:

$$P = \frac{\theta_J - \theta_A}{Z_{thJC} + Z_{thCA}} = 39 \text{ W}$$

c) For current impulses the transient thermal resistances have the values $Z_{thJC} = 1.2 \frac{K}{W}$ and $Z_{thCA} = 1.2 \frac{K}{W}$. From this results an allowed loss power of

$$P = \frac{\theta_J - \theta_A}{Z_{thJC} + Z_{thCA}} = 29 \text{ W}$$
3.4 Exercise 4

a) The temporal average value of the current is

\[ I_{AV} = \frac{1}{2\pi} \int_{0}^{2\pi} i_D(\omega t) \, d\omega t = \frac{1}{2\pi} \int_{0}^{\pi} i_D \sin(\omega t) \, d\omega t = \ldots = \frac{i_D}{\pi} \]

The RMS value of the current is

\[ I_{RMS} = \sqrt{\frac{1}{2\pi} \int_{0}^{2\pi} i_D^2(\omega t) \, d\omega t} = \sqrt{\frac{1}{2\pi} \int_{0}^{\pi} i_D^2 \sin^2(\omega t) \, d\omega t} = \ldots = \frac{i_D}{2} \]

Hence, the loss power in the diode can be calculated to

\[ P = V_S I_{AV} + r_D I_{RMS}^2 = 121 \text{ W} \]

b) The thermal equivalent circuit diagram can be seen in fig. 3.2.

c) The maximum allowed coolant temperature \( \vartheta_{A_{max}} \) is calculated as follows:

\[ \Delta \vartheta_{JA} = Z_{th,JA} \cdot P = (Z_{th,JC} + Z_{th,CA}) \cdot P = 90^\circ \text{C} \]

\[ \vartheta_{A_{max}} = \vartheta_{J_{max}} - \Delta \vartheta_{JA} = 25^\circ \text{C} \]

d) The limit of the temperature monitor can be calculated as follows:

\[ \vartheta_{C_{max}} = \vartheta_{J_{max}} - Z_{JC} \cdot P = 98^\circ \text{C} \]

e) The loss power can be calculated to

\[ P' = \frac{\vartheta_{J_{max}} - \vartheta_{A_{max}}}{Z_{th,JC} + Z_{th,CA}} = 108 \text{ W} \]
3.5 Exercise 5

a) The thermal equivalent circuit diagram can be seen in fig. 3.3.

![Thermal equivalent circuit diagram between junction and body of the diode](image)

Fig. 3.3: Thermal equivalent circuit diagram between junction and body of the diode

b) The time constants $\tau_{thi}(t)$ of the transient thermal resistances can be calculated to

$$\tau_{thi} = R_{thi} \cdot C_{thi}$$

The following values can be obtained:

$\tau_{th1} = 0.003 \text{s}$, $\tau_{th2} = 0.025 \text{s}$, $\tau_{th3} = 0.104 \text{s}$, $\tau_{th4} = 0.998 \text{s}$

c) The warming of the junction compared to the body can be calculated as follows:

$$\vartheta_A(t) = P \cdot \sum_{i=1}^{4} Z_{thi}(t) = P \cdot \sum_{i=1}^{4} R_{thi} \left(1 - e^{-\frac{t}{\tau_{thi}}}\right)$$

As the impulse duration is short, the body temperature can be assumed to be constant. The loss power impulse is decomposed into a positive power step of 800 W, beginning at $t = 0$ until $t = 5 \text{ ms}$ and into a negative power step of 500 W, beginning at $t = 5 \text{ ms}$ until $t = 35 \text{ ms}$. Both warmings are calculated separately and superposed afterwards. At the end of the impulse, at $t = 35 \text{ ms}$, the following can be obtained for the positive power step:

$$\vartheta_{J+} = 800 \text{ W} \left(0.019 \frac{\text{K}}{\text{W}} + 0.025 \frac{\text{K}}{\text{W}} + 0.063 \frac{\text{K}}{\text{W}} + 0.0023 \frac{\text{K}}{\text{W}}\right) = 87.5 \text{ K}$$

For the negative power step

$$\vartheta_{J-} = -500 \text{ W} \left(0.019 \frac{\text{K}}{\text{W}} + 0.023 \frac{\text{K}}{\text{W}} + 0.056 \frac{\text{K}}{\text{W}} + 0.002 \frac{\text{K}}{\text{W}}\right) = -50 \text{ K}$$

is obtained. Hence the junction warming at the end of the loss power impulse can be calculated to

$$\vartheta_J = \vartheta_{J+} + \vartheta_{J-} = 37.5 \text{ K}$$

d) The temperature profile during the impulse at the junction is shown in fig. 3.4.

e) It can be seen from the scaling of fig. 2.4 that the time constant $\tau_{thCA}$ between body and heatsink is much greater than the time constants $\tau_{thi}$ of the track between junction and body. Hence, for simplification one can use the transient thermal resistance $Z_{thCA}$ and the sum of the inner resistances $R_{thJC}$ for the calculations:

$$\vartheta_J = P \left(R_{thJC} + Z_{thCA}\right)$$
Fig. 3.4: Junction warming at the end of the loss power impulse

At the beginning of the process the junction is equivalent to the ambient temperature. Because of this the temperature rise can be calculated to

$$\Delta \vartheta_J = \vartheta_{J_{\text{max}}} - \vartheta_A = 80^\circ \text{C} - 35^\circ \text{C} = 45 \text{ K}$$

From this follows for the transient thermal resistance

$$Z_{thCA} = \frac{\Delta \vartheta_J}{P} - R_{thJC} = 0.187 \frac{\text{K}}{\text{W}}$$

Hence, a warming time of 100 s can be read from fig. 2.4.

f) In this case the assumption \( P = \text{const.} \) means continuous operation. For this mode of operation \( Z_{thCA} = 0.24 \frac{\text{K}}{\text{W}} \) can be read from fig. 2.4. The allowed warming is \( \Delta \vartheta_J = 90 \text{ K} \). The allowed loss power for continuous operation can be calculated to

$$P = \frac{\Delta \vartheta_J}{R_{thJC} + Z_{thJC}} = 155 \text{ W}$$