Grassmannian Product Codebooks for Limited Feedback FD-MIMO

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Full-Dimension MIMO Systems

- Utilize two-dimensional active antenna arrays to enable 2D-beamforming (azimuth, elevation)
- Allows to improve spectral efficiency, energy efficiency and network capacity
- We consider downlink linear multi-user MIMO transmission

\[ y_u = H_u^H F_u x_u + H_u^H \sum_{j=1, j \neq u}^U F_j x_j + z_u \]

- \( H_u \in \mathbb{C}^{N_t \times N_r} \), \( F_u \in \mathbb{C}^{N_t \times N_s} \): \( N_t \), \( N_r \) transmit and receive antennas, \( N_s \) streams per user
Full-Dimension MIMO Systems

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Limited Feedback Operation in FDD Systems

- Linear precoding requires channel state information (CSI) at the transmitter (CSIT)
- Time division duplex (TDD): uplink channel estimation can be employed
- Frequency division duplex (FDD): explicit limited feedback of CSI from the users
  - Downlink channel estimation and CSI quantization/feedback required
  - Downlink pilot overhead proportional to $N_t$
  - Uplink CSI overhead proportional to $N_t \cdot N_r$
- Hardly feasible for large $N_t$
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Two-Tier Precoding

- Split-up precoding into two-tiers:

\[ \mathbf{F}_u = \mathbf{F}_o \mathbf{F}_{i,u} \]

1. User-group-specific outer precoding \( \mathbf{F}_o \in \mathbb{C}^{N_t \times N_\ell} \)
2. User-specific inner precoding \( \mathbf{F}_{i,u} \in \mathbb{C}^{N_\ell \times N_s} \)

- \( \mathbf{F}_o \) often considered as wideband precoder (adapted based on channel statistics)
- \( \mathbf{F}_{i,u} \) often considered as subband precoder (adapted based on instantaneous CSI)
Two-Tier Precoding

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Two-Tier Precoding – Advantages/Disadvantages

- Reduced downlink pilot overhead and CSI feedback overhead
  - Infrequent estimation and feedback of the full channel matrix $H_u$
  - Per TTI estimation and feedback of $H_{o,u} = F_o^H H_u \in \mathbb{C}^{N_\ell \times N_t}, N_\ell \ll N_t$
- Reduced number of RF chains if $F_o$ is implemented in the RF domain (hybrid precoding)
  - Spatial multiplexing capabilities restricted to $N_\ell$
  - Suboptimal structure imposed on precoders
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Considered Two-Tier Precoding Strategy

- Outer-tier maximum eigenmode transmission (MET)
  - Maximum eigenmodes of the channel:
    \[
    H_u H_u^H = U_u \Sigma_u U_u^H,
    \]
    \[
    U_u^H U_u = I_{N_r}, \quad \Sigma_u = \text{Diag}(\sigma_u^{(1)}, \ldots, \sigma_u^{(N_r)})
    \]
  - Multi-user MET precoder:
    \[
    F_o = \begin{bmatrix}
    U_1^{(N_s)}, U_2^{(N_s)}, \ldots, U_{U_s}^{(N_s)}
    \end{bmatrix},
    \]
    \[
    U_j^{(N_s)} = \begin{bmatrix}
    u_j^{(1)}, \ldots, u_j^{(N_s)}
    \end{bmatrix}
    \]

- Inner-tier block-diagonalization (BD) precoding
  \[
  (U_o^{(N_s)})^H F_{i,u} = 0, \forall j \neq u,
  \]
  \[
  \text{rank} \left( (U_o^{(N_s)})^H F_{i,u} \right) = N_s
  \]

- \( U_o^{(N_s)} \) obtained from an SVD of \( H_{o,j} \in \mathbb{C}^{N_f \times N_r} \)
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- Inner-tier block-diagonalization (BD) precoding
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  \left(U_o^{(N_s)}\right)^H F_{i,u} \upharpoonright \neq 0, \ \forall j \neq u, \\
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- \(U_o^{(N_s)}\) obtained from an SVD of \(H_o,j \in \mathbb{C}^{N_\ell \times N_r}\)
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Full-Dimension MIMO Systems

Outer-Tier CSI Feedback

Conclusion
Many efficient CSI feedback methods are available for small-scale MIMO systems

These can be employed for the quantization and feedback of $\mathbf{H}_{o,u}$

We focus on computationally and rate-distortion efficient outer-tier CSI feedback of $\mathbf{H}_u$
• What information does the transmitter need about $H_u$ to enable MET precoding?

$$F_o = \left[ U_1^{(N_s)}, \ldots, U_U^{(N_s)} \right] \equiv \left[ U_1^{(N_s)}Q_1, \ldots, U_U^{(N_s)}Q_U \right],$$

$$Q_i^H Q_i = Q_i, Q_i^H = I_{N_s}$$

⇒ Subspace information is sufficient, since we do not consider power-loading over the eigenmodes

⇒ We apply Grassmannian quantization to $\text{span} \left( U_1^{(N_s)} \right)$ on the Grassmann manifold $G_{N_s}^{(N_t)}$
Grassmann Manifold Basics

- $\mathcal{G}_{N_s}^{(N_t)}$ manifold of $N_s$-dimensional subspaces of the $N_t$-dimensional real/complex Euclidean space
- Chordal distance between two subspaces defined by their orthogonal bases $U_1, U_2 \in \mathbb{C}^{N_t \times N_s}$

\[
d^2_G(U_1, U_2) = N_s - \text{tr} \left( U_1^H U_2 U_2^H U_1^H \right) = \sum_{i=1}^{N_s} \sin \varphi_i^2
\]

- $\varphi_i$ are the principal angles between the subspaces
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$$d_G^2 (U_1, U_2) = N_s - \text{tr} \left( U_1^H U_2 U_2^H U_1 \right) = \sum_{i=1}^{N_s} \sin \varphi_i^2$$

- $\varphi_i$ are the principal angles between the subspaces
The SNR-loss of MET-BD precoding with imperfect CSIT w.r.t. perfect CSIT is determined by the chordal distance CSI error $\Rightarrow$ minimum chordal distance quantization:

$$\hat{U}_{j,\text{single}}^{(N_s)} = \arg \min_{W_\ell \in Q^{(N_t)}_{N_s}} d_\delta^2 \left( U_{j}^{(N_s)}, W_\ell \right)$$

Quantization codebook $Q^{(N_t)}_{N_s} = \{ W_\ell \in \mathbb{C}^{N_t \times N_s} | W_\ell^H W_\ell = I_{N_s}, \forall \ell \}$ of size $2^b$

Rayleigh fading channels: random subspace quantization is asymptotically efficient

$$\bar{d}_{c,\text{single}}^2 = \mathbb{E} \left( d_\delta^2 \left( U_{j}^{(N_s)}, \hat{U}_{j,\text{single}}^{(N_s)} \right) \right) \propto 2^{-b/(N_s(N_t-N_s))}$$

Problem: huge codebooks are required for large $N_t \rightarrow$ computationally infeasible
Single-Stage Outer-Tier CSI Feedback

• The SNR-loss of MET-BD precoding with imperfect CSIT w.r.t. perfect CSIT is determined by the chordal distance CSI error ⇒ minimum chordal distance quantization:

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\hat{U}_{j,\text{single}}^{(N_s)} = \arg \min_{\mathbf{W}_\ell \in Q_{N_s}^{(N_t)}} d^2_G \left( \mathbf{U}^{(N_s)}_j, \mathbf{W}_\ell \right)
\]

• Quantization codebook \( Q_{N_s}^{(N_t)} = \{ \mathbf{W}_\ell \in \mathbb{C}^{N_t \times N_s} | \mathbf{W}_\ell^H \mathbf{W}_\ell = \mathbf{I}_{N_s}, \forall \ell \} \) of size \( 2^b \)

• Rayleigh fading channels: random subspace quantization is asymptotically efficient

\[
\bar{d}_{c,\text{single}}^2 = \mathbb{E} \left( d^2_G \left( \mathbf{U}^{(N_s)}_j, \hat{U}^{(N_s)}_{j,\text{single}} \right) \right) \propto 2^{-b/(N_s(N_t-N_s))}
\]

• Problem: huge codebooks are required for large \( N_t \) → computationally infeasible
Multi-Stage Outer-Tier CSI Feedback

- Employ a product-codebook construction to reduce the quantization complexity

\[ \hat{U}_1 \in Q_{1,S}^{(N_t)} \subset \mathbb{C}^{N_t \times S}, \quad \hat{U}_2 \in Q_{2,N_s}^{(S)} \subset \mathbb{C}^{S \times N_s} \]

- The second-stage quantization codebook \( Q_{2,N_s}^{(S)} \) should be independent of \( \hat{U}_1 \)
Employ a product-codebook construction to reduce the quantization complexity

\[ \hat{U}_1 \in Q^{(N_t)}_{1,S} \subset \mathbb{C}^{N_t \times S}, \quad \hat{U}_2 \in Q^{(S)}_{2,N_s} \subset \mathbb{C}^{S \times N_s} \]

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Multi-Stage Outer-Tier CSI Feedback (II)

- First-stage quantizer:
  \[
  \hat{\mathbf{U}}_1 = \arg \min_{\mathbf{W}_\ell \in Q_{1,S}^{(N_t)}} \| \mathbf{d}_G^2 \left( \mathbf{U}_j^{(N_s)}, \mathbf{W}_\ell \right) \|_{1} \in \mathbb{C}^{N_t \times S}
  \]

- Subspace-quantization based combining
  \[
  \mathbf{B} = \hat{\mathbf{U}}_1^H \mathbf{U}_j^{(N_s)} \left( \left( \mathbf{U}_j^{(N_s)} \right)^H \hat{\mathbf{U}}_1 \hat{\mathbf{U}}_1^H \right)^{-\frac{1}{2}} \in \mathbb{C}^{S \times N_s},
  \]
  \[
  \mathbf{B}^H \mathbf{B} = \mathbf{I}_{N_s}, \quad \| \mathbf{d}_G^2 \left( \hat{\mathbf{U}}_1, \mathbf{U}_j^{(N_s)} \right) \|_{1} = \| d_2^2 \left( \hat{\mathbf{U}}_1 \mathbf{B}, \mathbf{U}_j^{(N_s)} \right) \|_{1}
  \]

- Second-stage quantizer:
  \[
  \hat{\mathbf{U}}_2 = \arg \min_{\mathbf{W}_\ell \in Q_{2,N_s}^{(S)}} \| \mathbf{d}_G^2 \left( \mathbf{B}, \mathbf{W}_\ell \right) \|_{1} \in \mathbb{C}^{S \times N_s}
  \]

- Total quantized CSI: 
  \[
  \hat{\mathbf{U}}_{j,\text{dual}}^{(N_s)} = \hat{\mathbf{U}}_1 \hat{\mathbf{U}}_2 \in \mathbb{C}^{N_t \times N_s}
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Multi-Stage Outer-Tier CSI Feedback (II)

- First-stage quantizer:
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  \hat{U}_1 = \arg \min_{W_{\ell} \in Q_{1,S}^{(N_t)}} d_G^2 \left( \left( W_{\ell} \right), \left( U_{j}^{(N_s)} \right) \right) \in \mathbb{C}^{N_t \times S}
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  \[
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Multi-Stage Outer-Tier CSI Feedback (II)

- First-stage quantizer:
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  \[ B^H B = I_{N_s}, \quad d_G^2 \left( \hat{U}_1, U_j^{(N_s)} \right) = d_G^2 \left( \hat{U}_1 B, U_j^{(N_s)} \right) \]

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- Total quantized CSI:
  \[
  \hat{U}_{j,\text{dual}}^{(N_s)} = \hat{U}_1 \hat{U}_2 \in \mathbb{C}^{N_t \times N_s}
  \]
The product codebook quantization error is determined by:

$$\tilde{d}_{c,dual}^2 = \mathbb{E} \left( d_G^2 \left( \hat{U}_{j,(N_s)}, \hat{U}_{j,dual}^{(N_s)} \right) \right) = N_s - \frac{1}{N_s} \left( N_s - \tilde{d}_{c,1}^2 \right) \left( N_s - \tilde{d}_{c,2}^2 \right)$$

Notice $2^{64} + 2^{64} = 2^{65} \ll 2^{128} \rightarrow$ significant complexity reduction
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Notice \( 2^{64} + 2^{64} = 2^{65} \ll 2^{128} \rightarrow \text{significant complexity reduction} \)
Multi-Scattering Directional Channels

- Multi-scattering directional channel model:

\[ H_u = \sqrt{N_t N_r} \sum_{p=1}^{N_p} \alpha_{p,u} a_t(\phi_{p,u}) a_r(\theta_{p,u})^T \]

- Antenna array response vector of a uniform linear array:

\[
\begin{bmatrix}
 a_{t,ULA}^U(\phi)
\end{bmatrix}_k = \frac{g_e(\phi)}{\sqrt{N_t}} \exp \left( j \frac{2\pi d}{\lambda} (k - 1) \sin(\phi) \right)
\]
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\[ a^{ULA}_t(\phi)_k = g_e(\phi) \frac{\exp\left(j \frac{2\pi d}{\lambda} (k - 1) \sin(\phi)\right)}{\sqrt{N_t}} \]
Multi-scattering directional channels: DFT-based codebooks perform well for small $b$

$$Q_{S,DFT}^{(N_t,N_{DFT})} = \left\{ \frac{1}{\sqrt{N_t}} [D_{N_{DFT}}]_{1:N_t,S} | S, |S| = S \right\}, \quad [D_{N_{DFT}}]_{\ell,k} = e^{-j2\pi(\ell-1)(k-1)/N_{DFT}}$$

- Oversampled DFT codebooks exhibit an error saturation for $N_p \geq 1$ due to unit-modulus constraint
- Example: $N_t = 64$, $N_r = N_s = 2$, $N_p = 4$
Quantization of Multi-Scattering Directional Channels

• Multi-scattering directional channels: DFT-based codebooks perform well for small $b$

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[Diagram showing quantization distortion vs. quantization bits for different values of $S$ and $b$.]
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Throughput Performance

- Measured channel traces of Nokia Bell Labs, Stuttgart
- $N_t = 64$, $N_r = N_s = 2$, $U = 6$ users with velocities between 15 and 25 km/h @2 GHz center frequency

\[
R_f R_t \approx 5.36 \text{ bit/ms/MHz.}
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Throughput Performance

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- $N_t = 64$, $N_r = N_s = 2$, $U = 6$ users with velocities between 15 and 25 km/h @2 GHz center frequency

\[
R_f R_t \approx 5.36 \text{ bit/ms/MHz}.
\]
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$$\left( \log_2 \left( \left( \frac{N_{\text{DFT}}}{S} \right) + S N_s b_s \right) \right) R_f R_t \approx 5.36 \text{ bit/ms/MHz}.$$
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Summary and Conclusion

• Grassmannian dual-stage product codebooks can perform very close to single-stage quantization when employing RVQ/RSQ

• Significant complexity reduction by utilizing two relatively small codebooks rather than a single large codebook

• In multi-scattering channels, the product codebook can additionally mitigate the error floor of DFT codebooks

• Future work: further extension to multi-stage product codebooks?