Spectra as Performance Metrics for Fiber-Optic Communication System Design

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Motivation

- Recent efforts in community to provide information-theoretic tools for design of fiber-optic communication systems.
- Nice summary in

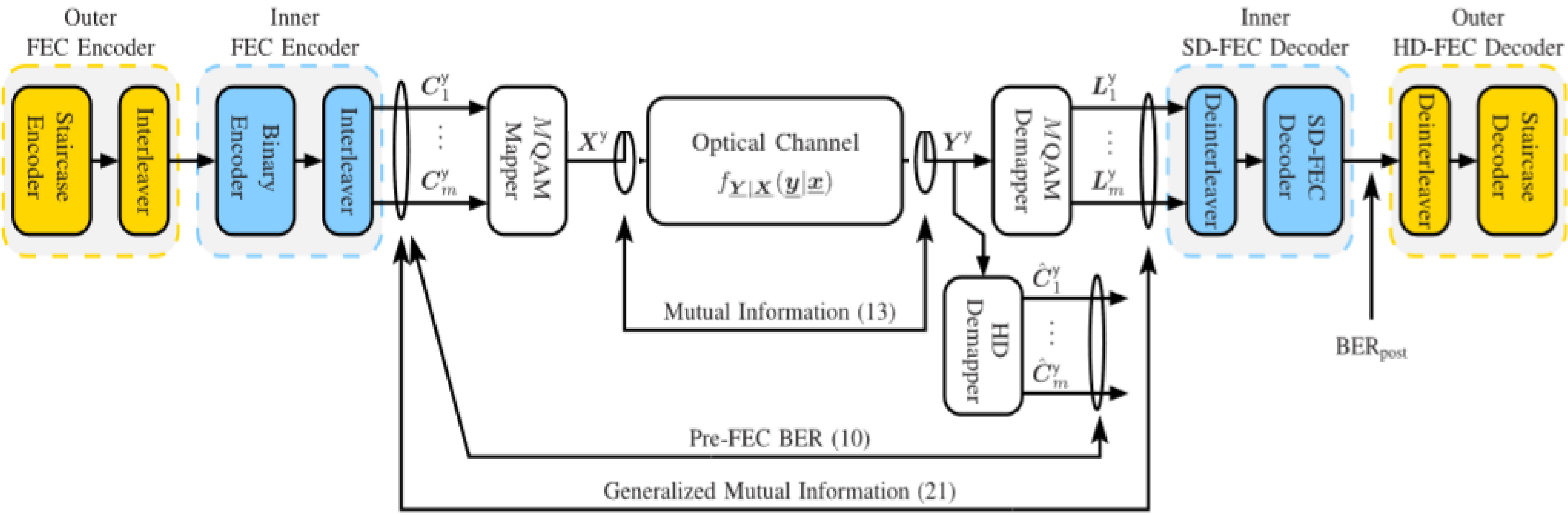
  L. Schmalen "**Performance Metrics for Communication Systems with Forward Error Correction**," ECOC 2018.

➔ **Discuss current state and possible extensions**.

Outline

- **Thresholds as performance metrics**
- **Limitations of BER threshold**
- **BER spectrum**
- **Uncertainty spectrum**
- **Conclusions**

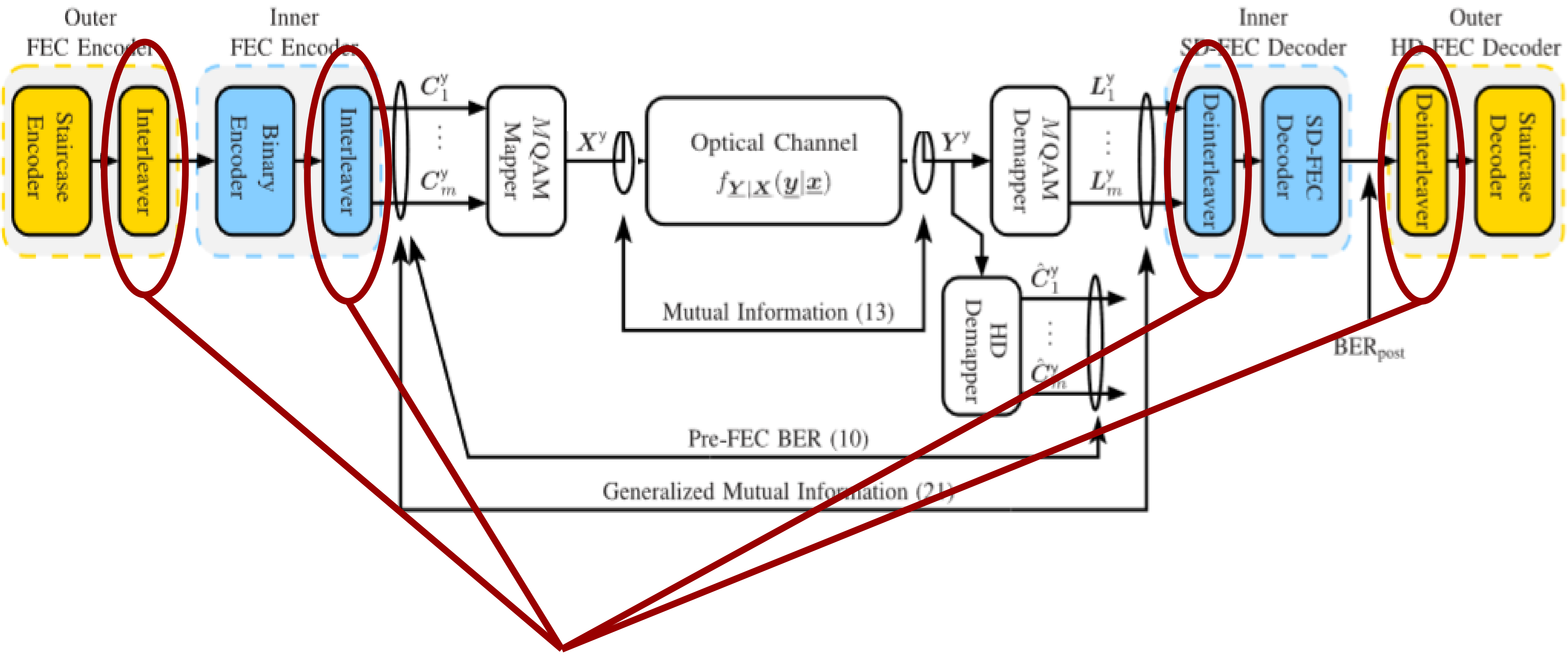
Design by Thresholds



- Interfaces between components are defined by thresholds
  - Bit error rate (BER)
  - Mutual information
  - Generalized mutual information
  - GMI, NGMI, ABC, AIR, RBMD,…….
- **Components are designed to respect thresholds**

Award-winning paper:
A. Alvarado et al "**Replacing the Soft-Decision FEC Limit Paradigm in the Design of Optical Communication Systems**," JLT, Vol 34, No 2, 2016.

Design by Thresholds

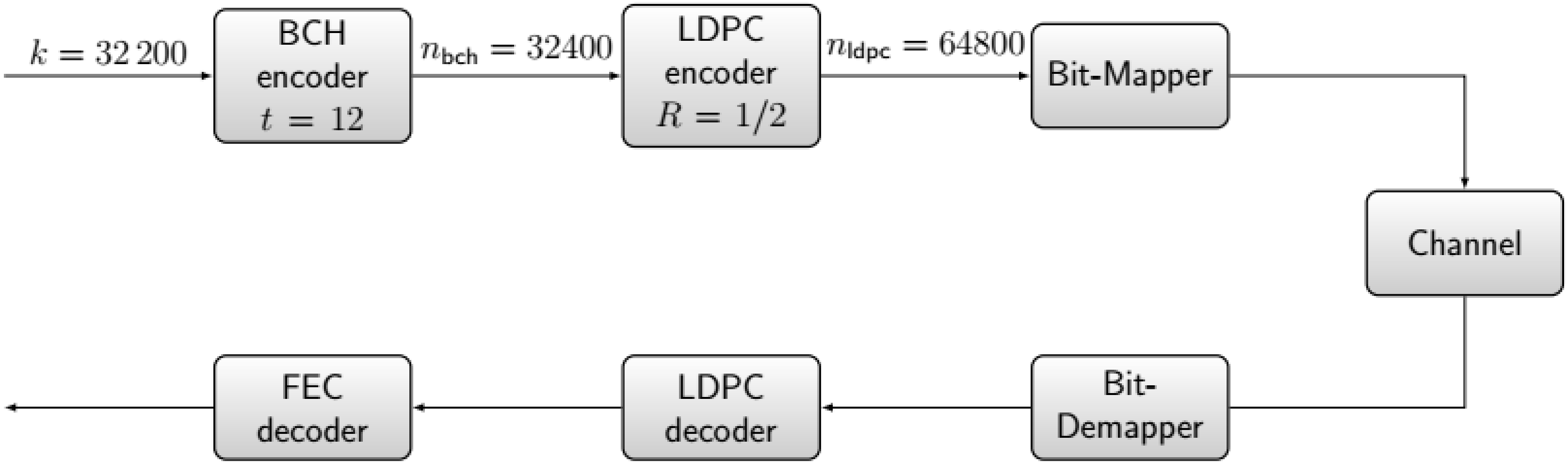


➔ Crucial Assumption: 'Infinite' Interleavers.

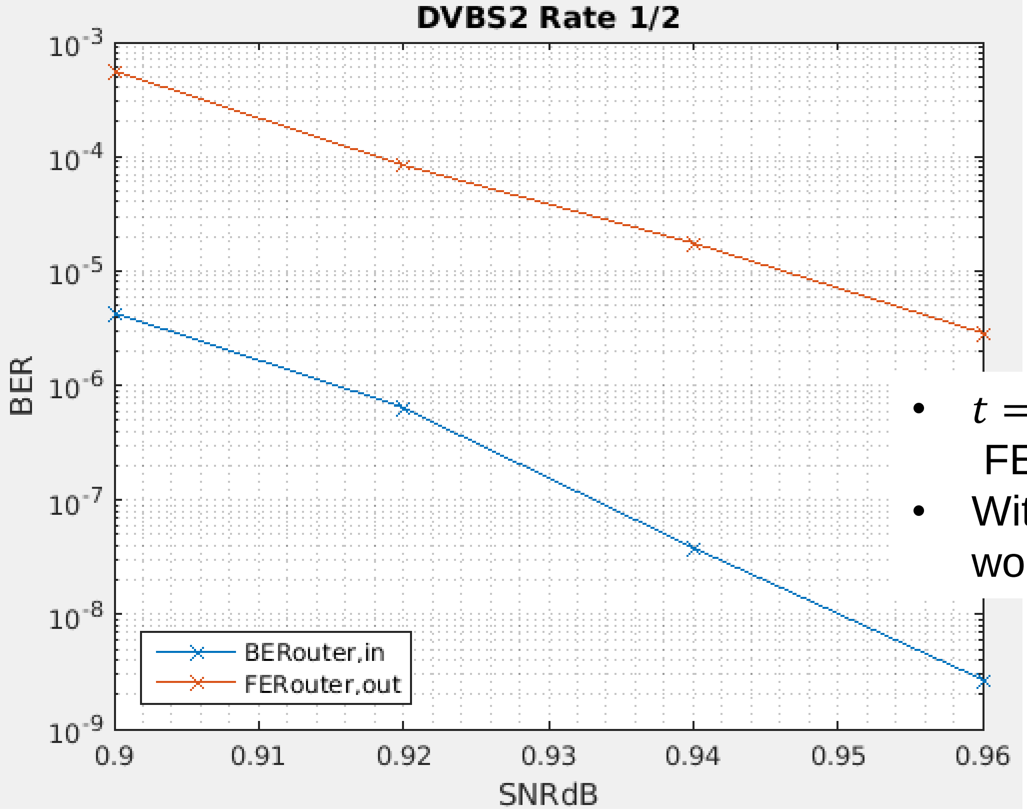
➔ Translates into latency in practical transceivers.

What if there is no ∞-interleaver?

Example: DVB-S2



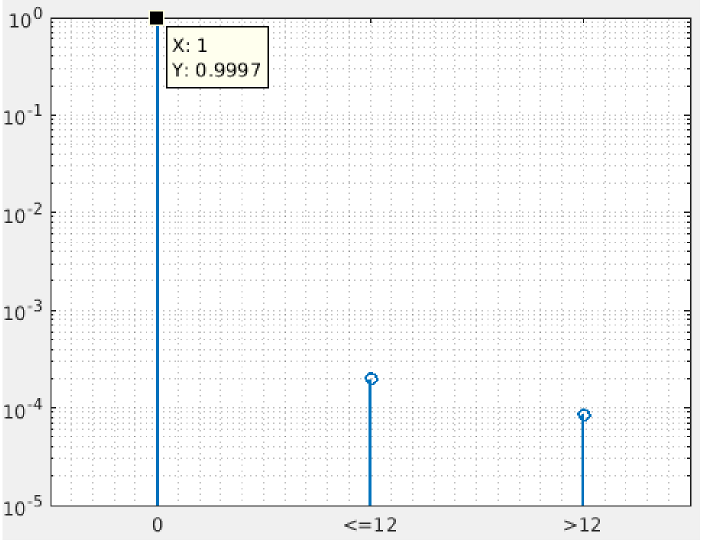
Simulation Results



- $t = 12$ BCH outer code achieves FER = 3×10^{-6} at 0.96 dB.
- With ∞-interleaver, $t = 1$ BCH outer code would achieve FER= 4×10^{-9} at 0.96 dB.

➔**BER thresholds provide only limited insights for design**

Design by BER Spectrum



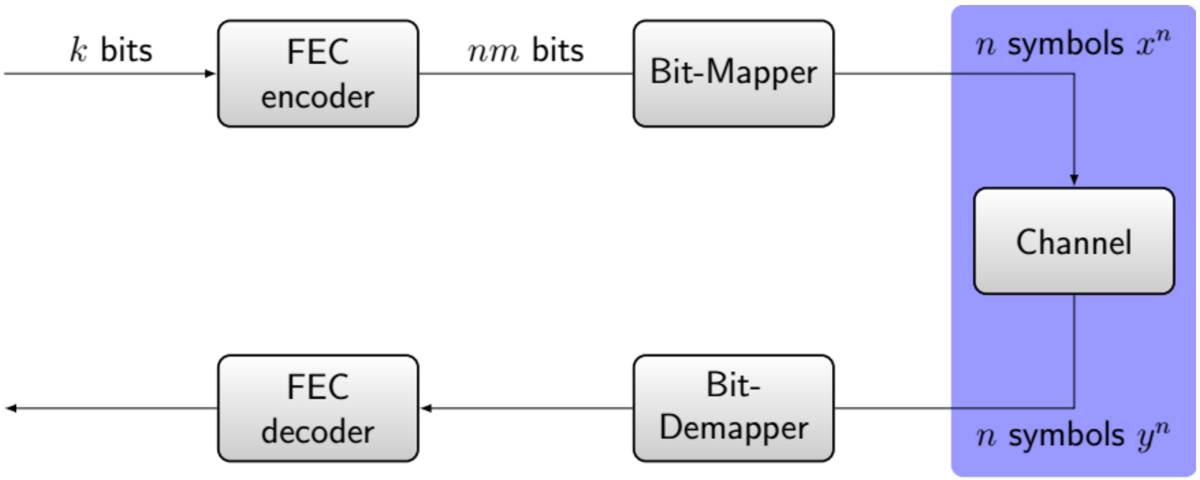
- Quantify inner code performance by **BER Spectrum**, i.e., the statistics of the fraction of erroneous bits per frame.
- Use BER spectrum for design:
  - For given t, design inner code with constraint $\mathrm{Pr}(\#\mathrm{errors} > \mathrm{t}) <$ target FER.
  - For given inner code, choose t so that $\mathrm{Pr}(\#\mathrm{errors} > \mathrm{t})$ <target FER.

Soft-Decision FEC: Uncertainty*

***Based on Gallager's error exponent, details on uncertainty in**

[1] G. Böcherer, P. Schulte, and F. Steiner, "Probabilistic Shaping and Forward Error Correction for Fiber-Optic Communication Systems," *J. Lightw. Technol., 2019.*

Setup



- **FEC Overhead:** $\mathrm{OH}_{\mathrm{FEC}} = 1 - \frac{k}{nm}$

For a **given channel**, what FEC-OH is achievable?

Achievable FEC Overhead

- We don't know the exact channel, but we have a **measurement** x^n, y^n (*e.g.*, of a QAM signal)
- How large FEC Overhead $1 - \frac{k}{mn}$ is required to recover x^n from y^n by FEC decoding?

Achievable FEC Overhead

Decoding metric, e.g., $P_{X|Y}^{n}(x^n|y^n)$

- **Uncertainty**

$$U_q(x^n, y^n) = -\frac{1}{n}\log_2 \frac{q(x^n, y^n)}{\sum_{a^n \in \mathcal{X}^n} q(a^n, y^n)}$$

- **Theorem**

$$\Pr(\hat{W} \neq w_0 | C^n(w_0) = x^n, Y^n = y^n) \leq 2^{-nm\left[1 - \frac{k}{nm} - \frac{U_q(x^n, y^n)}{m}\right]}$$

Fraction of FEC codes that cannot decode x^n from y^n

$$\begin{aligned}
&\Pr(\hat{W} \neq 1 | X^n = x^n, Y^n = y^n) \\
&\leq \Pr\left[\sum_{w=2}^{|\mathcal{C}|} L(w) \geq 1 \middle| X^n = x^n, Y^n = y^n\right] \\
&\leq \mathbb{E}\left[\sum_{w=2}^{|\mathcal{C}|} L(w) \middle| X^n = x^n, Y^n = y^n\right] \\
&= q(x^n, y^n)^{-1}\,\mathbb{E}\left[\sum_{w=2}^{|\mathcal{C}|} q(C^n(w), y^n)\right] \\
&= (|\mathcal{C}| - 1) q(x^n, y^n)^{-1}\,\mathbb{E}\left[q(C^n, y^n)\right] \\
&\leq |\mathcal{C}| q(x^n, y^n)^{-1}\,\mathbb{E}\left[q(C^n, y^n)\right] \\
&= 2^{nmR_{fec}} q(x^n, y^n)^{-1} \sum_{a^n \in \mathcal{X}^n} |\mathcal{X}|^{-n} q(a^n, y^n) \\
&= 2^{-nm\left(1 - \frac{1}{nm}\log_2 \frac{q(x^n, y^n)}{\sum_{a^n \in \mathcal{X}^n} q(a^n, y^n)} - R_{fec}\right)}
\end{aligned}$$

Interpretation

FEC Overhead

$$2^{-nm\left[1-\frac{k}{nm}-\frac{U_q(x^n,y^n)}{m}\right]}$$

FEC Rate

Uncertainty

- For FEC overhead larger than uncertainty, exponent is negative.
- Backing-off in FEC rate leads to exponential decay of probability to pick a bad code.
- **Uncertainty identifies the phase transition to the possible.**

Works for any measurement x^n, y^n without any further assumptions.

From Measurement Property to Channel Property

$$U_q(x^n, y^n) = -\frac{1}{n} \log_2 \frac{q(x^n, y^n)}{\sum_{a^n \in \mathcal{X}^n} q(a^n, y^n)}$$

$n \to \infty$, ergodicity

Channel property:
Achievable FEC overhead $\frac{U_q}{m}$

$n \approx$FEC block length, many pairs x^n, y^n

Channel property:
Uncertainty spectrum

Put forward in:

- Essiambre et al "**Capacity Limits of Optical Fiber Networks**", JLT 2010.
- Alvarado et al "**Achievable Information Rates for Fiber-Optics: Applications and Computations**," JLT 2018.

Example

Note: finite-length informationi theory accounts for variance, e.g., Polyankiy et al "**Channel coding rate in the finite blocklength regime**," ITT 2010.

- QPSK Measurement with n =10 million bits at 1 dB SNR.
- FEC block length n_{FEC} =10 000.

Threshold

➔ Use all 10 million samples to estimate uncertainty $U_q = 0.4373$ bits

➔ FEC overhead 0.4373 bits is achievable, asymptotically in the blocklength.

➔ blocklength is finite, we have to back-off, **how much**?

$$U = 0.4373$$

Spectrum

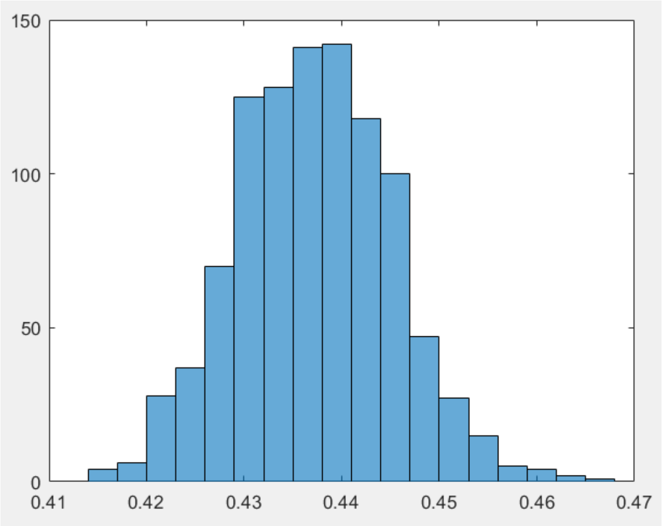
➔ Split measurement in $\frac{n}{n_{\text{FEC}}} = 1000$ chunks.

➔ Calculate 1000 uncertainties.

➔ Plot the spectrum

➔ Back-off to FEC overhead 0.47 bits.

➔ Corresponds to **0.42 dB** back-off in SNR**.**



Conclusions

Observations

- Spectra account for finite-length effects.
- Spectra account for correlations created by components.
  ➔ Alternative to threshold and ∞-interleaver.

Possible Directions

- New criteria for design of concatenated components?
- To design components with desired spectra, can we borrow from rate-distortion theory or information-theoretic security?
- Shorter interleaver, better systems?

FEC Code Ensemble

- **FEC Code**

$$C = \{C^n(1), C^n(2), \dots, C^n(2^k)\}$$

with symbols $C_i(w)$ in the channel alphabet (*e.g.*, 16-QAM.)

- **Encoder:** map k bits w to code word $x^n(w)$.
- **Decoder:**

$$\hat{w} = \underset{w \in \{1,2,\dots,2^k\}}{\operatorname{argmax}} \; q(c^n(w), y^n)$$

Decoding metric

- **Error probability:***

$$\Pr(\hat{W} \neq w_0 | C^n(w_0) = x^n, Y^n = y^n)$$

*=fraction of FEC codes that cannot decode x^n from y^n.