Decoder-in-the-Loop: Genetic Optimization-based Code Design

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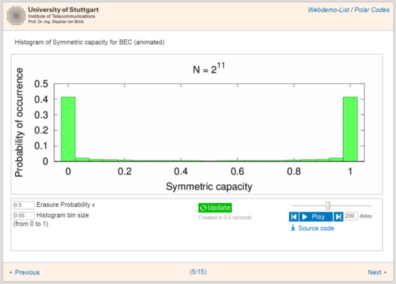
Outline

Agenda

1. Introduction
2. Genetic Algorithm-based Polar Code Construction
3. Results for different decoders and channels
4. Decoding Complexity Reduction
5. Genetic Algorithm-based LDPC Code Design
6. Summary

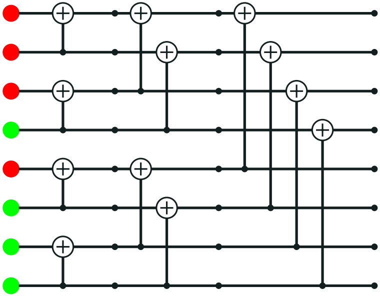
Polar Codes



- Polar codes were introduced by Arıkan.
- Asymptotically achieve capacity under SC decoding.
- Low encoding and SC decoding complexity $\mathcal{O}(N \log N)$.
- Based on the concept of channel polarization.
  - Uncoded information bits are transmitted over the reliable (noiseless) bit channels.
  - Frozen (known) bits are transmitted over the unreliable (noisy) bit channels.

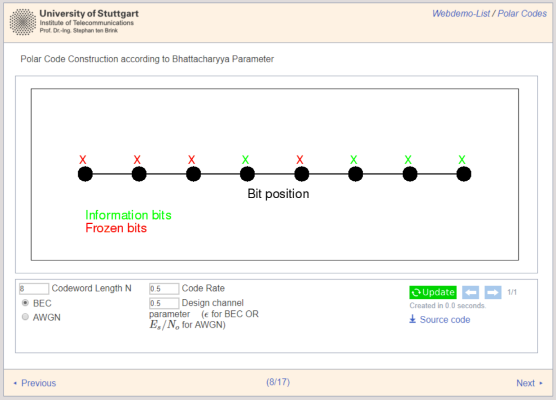
`http://webdemo.inue.uni-stuttgart.de/webdemos/08_research/polar/`

Polar Decoding



- SC decoder: achieves capacity for infinite length codes.
- Belief propagation (BP) decoder: better BER performance than SC for finite length codes.
- Successive cancellation list (SCL) decoder: approaches the ML decoder performance.
- SCL decoding of the modified polar code (outer high rate CRC code concatenated with inner polar code): outperforms the ML decoder of pure polar codes.

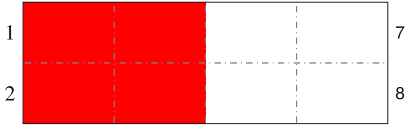
Polar Code Construction



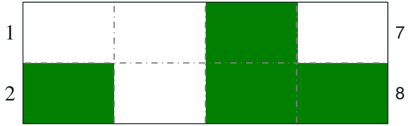
- Finding the best k bit positions for information transmission.
  - The remaining $N-k$ bit positions are frozen.
- State-of-the-art design methods assume SC decoding.
  - Thus, not necessarily optimal under BP and SCL decoding.
  - Decoder-tailored code design will enhance the performance!

Frozen/non-frozen set "Frozen Channel Chart"

Bhattacharyya-based design $\mathbf{A} = [0\,0\,0\,1\,0\,1\,1\,1]$ (i.e., $R_c = 0.5$)



Random design $\mathbf{A} = [1\,0\,1\,0\,1\,0\,1\,0]$ (i.e., $R_c = 0.5$)



- For the $\mathscr{P}$(8,4)-code, the information set $\mathbb{A} = \{4,6,7,8\}$
  - can be represented as $\mathbf{A} = [0\,0\,0\,1\,0\,1\,1\,1]$
- Code rate $R_c = \frac{\sum \mathbf{A}}{N}$
- Bit-channels are sorted with decreasing Bhattacharyya values. Colored: frozen "0"; white: non-frozen "1"

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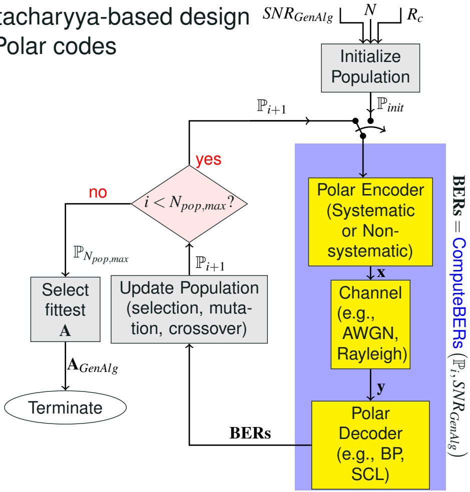
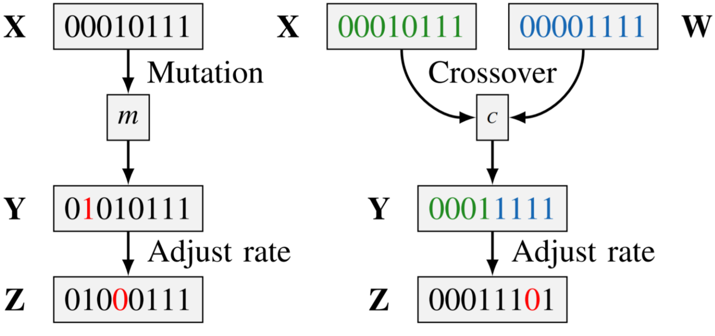
Abstract view

- Parameters for optimization
- Initial population, e.g.:
  - Bhattacharyya-based design
  - RM-Polar codes



Evolutionary Transformations



(a) Mutation or swapping (b) Crossover

- Code rate maintained (i.e., stayes fixed)

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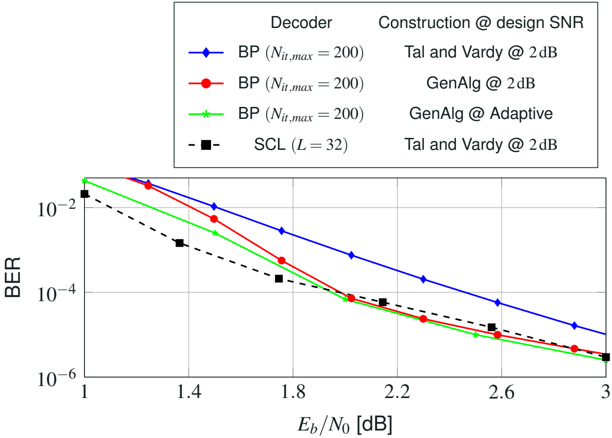
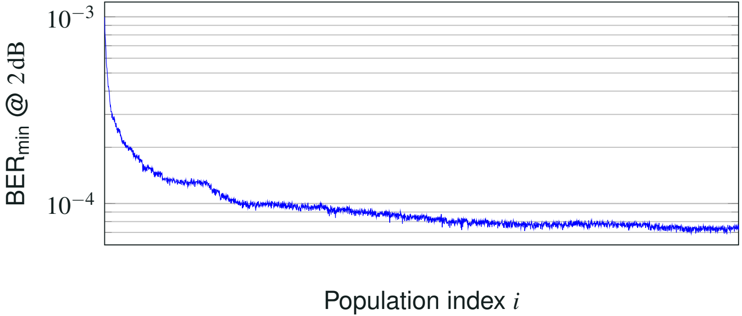
BP-tailored Polar Codes



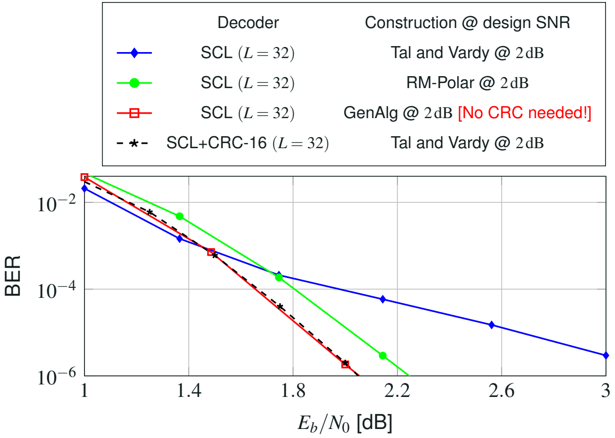
- GenAlg-based construction of a $\mathscr{P}(2048,1024)$-code under BP ($N_{it,max} = 200$) decoding over the AWGN channel and no CRC is used.

Epochs of Genetic Optimization



- Evolution of the BER at SNR_{GenAlg} $(E_b/N_0) = 2\,\mathrm{dB}$.
- each code candidate was simulated to count at least 1000 bit errors

SCL-tailored Polar Codes



- BER performance of the GenAlg-based $\mathscr{P}(2048,1024)$-code under SCL decoding over the AWGN channel.
- Note that the CRC-aided polar code (- ⋆ -): $N = 2048$, $k = 1024$, $r = 16$, $R_c = 0.5$ and, thus, the polar code is a $\mathscr{P}(2048,1040)$-code.

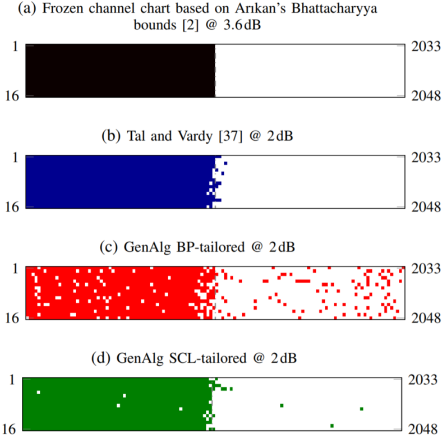
Decoder-tailored Polar Codes

Construction @ design SNR	Decoder to reach BER 10^{-4}		
	SC	BP $(N_{it,max} = 200)$	SCL $(L = 32)$
Bhattacharyya @ $3.6\,\mathrm{dB}$	**2.7 dB**	2.45 dB	1.8 dB
Tal and Vardy @ $2\,\mathrm{dB}$	**2.65 dB**	2.45 dB	2 dB
GenAlg BP-tailored @ $2\,\mathrm{dB}$	$>$ 9 dB	**2 dB**	$>$ 7 dB
GenAlg SCL-tailored @ $2\,\mathrm{dB}$	$>$ 6 dB	2.55 dB	**1.65 dB**

- Illustration of polar design and decoder architecture mismatch by evaluating the minimum E_b/N_0 required to achieve a target BER of 10^{-4} for a $\mathscr{P}$(2048,1024)-code over AWGN channel

Frozen Channel Chart

- Frozen bit position pattern of a $\mathscr{P}(2048,1024)$-code with different polar code construction algorithms. The 2048 bit positions are plotted over a 16×128 matrix. Note that the bit-channels are sorted with decreasing Bhattacharyya parameter value. Colored: frozen; White: non-frozen.



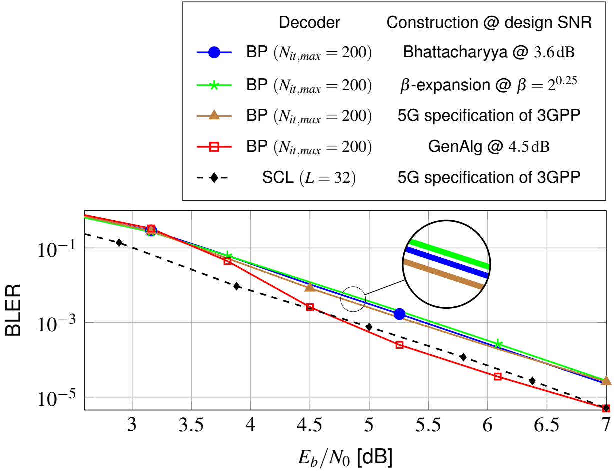
Weight enumerators

Construction @ design SNR	d_{min}	A_8	A_{16}
Tal and Vardy @ $2\,\mathrm{dB}$	16	0	11648
GenAlg BP-tailored @ $2\,\mathrm{dB}$	8	8	773
GenAlg SCL-tailored @ $2\,\mathrm{dB}$	16	0	1

- Using the algorithm described in [2]
- GenAlg reduces number of low-weight codewords!

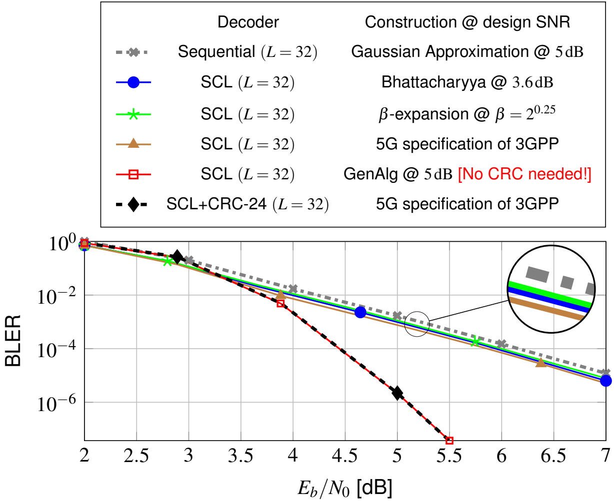
[2] B. Li, H. Shen, and D. Tse, "An Adaptive Successive Cancellation List Decoder for Polar Codes with Cyclic Redundancy Check," IEEE Commun. Lett., Dec. 2012.

BP-tailored Polar Codes



- BLER performance of the GenAlg-based $\mathscr{P}(1024{,}512)$-code under BP decoding over the Rayleigh fading channel and no CRC is used.

SCL-tailored Polar Codes



- BLER performance of the GenAlg-based $\mathscr{P}$(1024,512)-code under SCL decoding over the Rayleigh fading channel.
- Note that the CRC-aided polar code (-◆-): $N = 1024$, $k = 512$, $r = 24$, $R_c = 0.5$ and, thus, the polar code is a $\mathscr{P}$(1024,536)-code.

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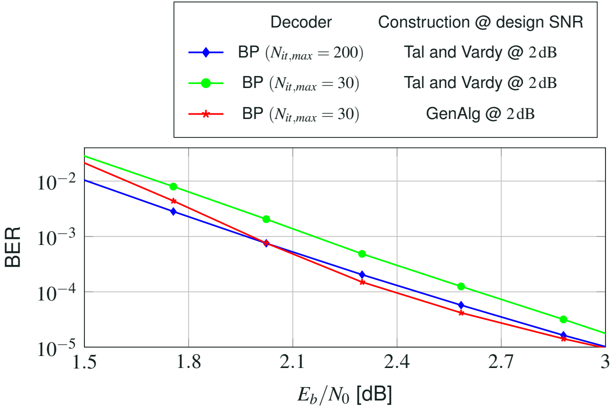
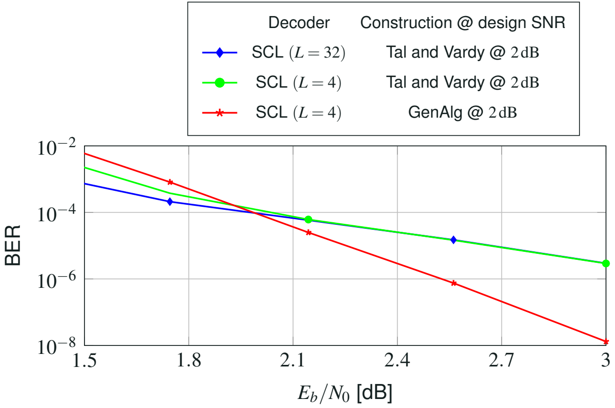
BP Decoding



- BER performance of the GenAlg-based $\mathscr{P}(2048,1024)$-code under BP decoding with reduced $N_{it,max}$ over the AWGN channel and no CRC is used.

SCL Decoding



- BER performance of the GenAlg-based $\mathscr{P}(2048,1024)$-code under SCL decoding with reduced list size L over the AWGN channel and no CRC is used.

For more details

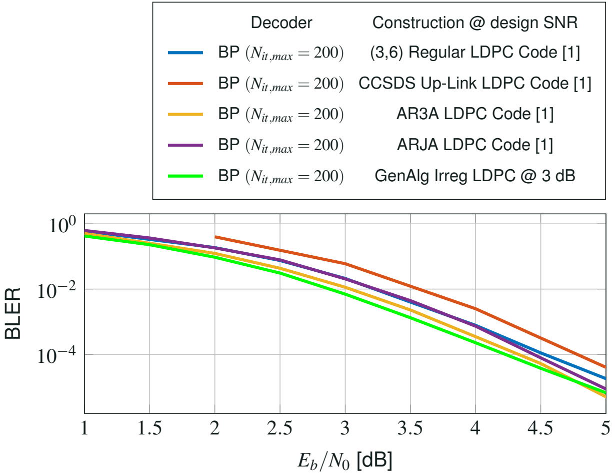
- A. Elkelesh, M. Ebada, S. Cammerer, and S. ten Brink, "Decoder-tailored Polar Code Design Using the Genetic Algorithm," ArXiv e-prints, Jan. 2019. A poster today!
- A. Elkelesh, M. Ebada, S. Cammerer, and S. ten Brink, "Genetic Algorithm-based Polar Code Construction for the AWGN Channel," in IEEE Inter. ITG Conf. on Syst., Commun. and Coding (SCC), Feb. 2019.
- `https://github.com/AhmedElkelesh/Genetic-Algorithm-based-Polar-Code-Construction`

Agenda

LDPC code design

- We design the whole parity-check matrix (i.e., $\mathbf{H}$-matrix)
  - No degree profile optimization (e.g., EXIT charts)
  - No PEG algorithm used

Results $(N = 128, R_c = 0.5)$



- No special graph structure

[1] G. Liva et. al, “Code Design for Short Blocks: A Survey,” ArXiv e-prints, Oct. 2016.

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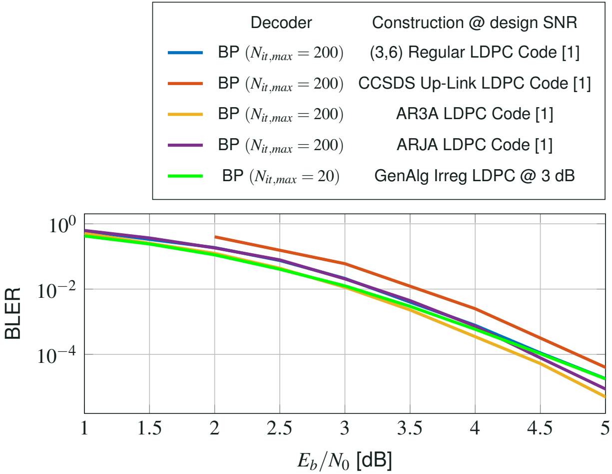

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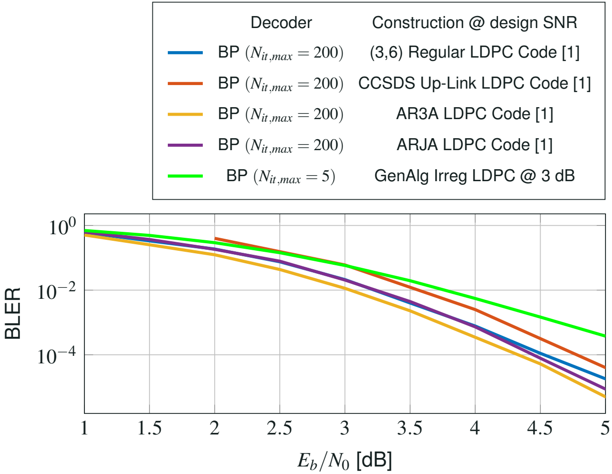

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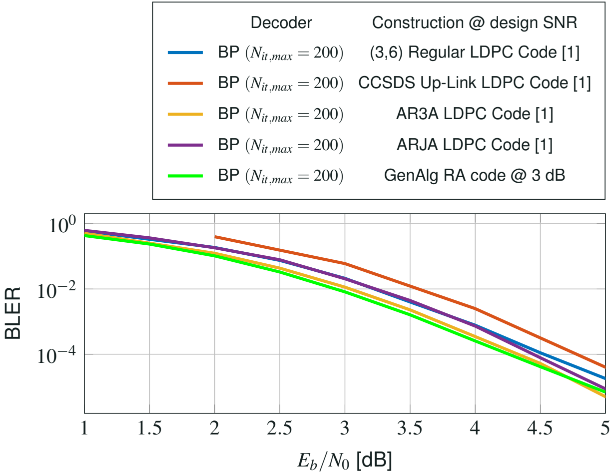

- RA graph structure
  - Similar to LDPC codes from DVB-S.2 standard

[1] G. Liva et. al, “Code Design for Short Blocks: A Survey,” ArXiv e-prints, Oct. 2016.

For more details

- A. Elkelesh, M. Ebada, S. Cammerer, L. Schmalen, and S. ten Brink, “Decoder-in-the-Loop: Genetic Optimization-based LDPC Code Design,” submitted/under review, Feb. 2019.
- `https://github.com/AhmedElkelesh/Link-will-be-Available-After-Review`

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Summary

- New polar code construction algorithm
  - the resulting codes are decoder-tailored and channel-tailored
  - BP-tailored Polar Codes
  - SCL-tailored Polar Codes
  - outperforms the state-of-the-art construction algorithms
- Codes can be designed with the aim of reducing the decoding complexity
- Can be used to design LDPC codes
  - designing the $\mathbf{H}$-matrix
  - no EXIT curves matching
  - no PEG used

Thank you for your attention!

Backup Slide (Reference polar codes)

- [R1] I. Tal and A. Vardy, "How to Construct Polar Codes," IEEE Trans. Inf. Theory, vol. 59, no. 10, pp. 6562–6582, Oct. 2013.
- [R2] B. Li, H. Shen, and D. Tse, "A RM-Polar Codes," ArXiv e-prints, July 2014.
- [R3] "Technical Specification Group Radio Access Network," 3GPP, 2018, TS 38.212 V.15.1.1. [Online]. Available: http://www.3gpp.org/ftp/Specs/archive/38 series/38.212/
- [R4] P. Trifonov, "Efficient Design and Decoding of Polar Codes," IEEE Trans. Commun., vol. 60, no. 11, pp. 3221–3227, Nov. 2012.
- [R5] P. Trifonov, "Design of Polar Codes for Rayleigh Fading Channel," in Inter. Symp. Wireless Commun. Syst. (ISWCS), Aug. 2015, pp. 331–335.
- [R6] G. He, J. C. Belfiore, I. Land, G. Yang, X. Liu, Y. Chen, R. Li, J. Wang, Y. Ge, R. Zhang, and W. Tong, "β-expansion: A Theoretical Framework for Fast and Recursive Construction of Polar Codes," in IEEE Global Commun. Conf. (GLOBECOM), Dec. 2017, pp. 1–6.

References